Notation. Γ, circulation; θ, polar angle of the cylindrical coordinate system; Φ_c, velocity potential of the main flow; Φ, velocity potential; κ, curvature of the surface; $r = 1 + \xi(z, t)$, equation of the surface of the capillary jet; T, surface tension; α, wave number; α_c, growth factor; Q, surface charge; δ_c, initial amplitude of sinusoidal perturbation of the surface of the capillary jet; α_0, wave number bounding the region of wave-number instability; λ, curvature of the initial sawtooth perturbation; N, number of harmonics; t_d, time of decay; t_*, time during which the surface charge changes abruptly; a, radius of jet; q_1, surface charge per unit length of the jet at the given moment of time; n, external normal to the surface of the jet; α_m, wave number corresponding to the maximum value of α_c.

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EFFECT OF FRICTION ON LOW FREQUENCY SOUND PROPAGATION IN A GAS–LIQUID FOAM

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A model is proposed for propagation of low frequency acoustic disturbances in a gas–liquid foam with consideration of friction on interphase boundaries during liquid motion in a system of interconnected microcapillaries. A Burgers equation with quasilinear convolution-type term is obtained. Structure and dynamics of linear signals are studied over the range of applicability of the model.

The spectrum of technological processes which employ foams and foamlike structures has expanded precipitously and currently encompasses a most varied range of applications [1]. To support production techniques involving foams both in cases where foam formation must be intensified, and in situations where foam disrupts the normal course of a process, a precise realtime knowledge of foam parameters is required. Since one method of solving such problems involves acoustical diagnostics, the problem of determining sound propagation characteristics in foam arises.

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The present study will analyze hydrodynamic effects which affect the evolution of an acoustical disturbance in foam.

Such an unusual state as foam develops in a two-phase medium due to the presence in the liquid of surface active materials (SAM), which lead to a reduction in the surface tension coefficient [1]. Due to their clearly expressed dipole structure SAM molecules concentrate on the interphase boundary, arranging themselves so that the hydrophobic end of the molecule is turned toward the gas, while the hydrophilic end is submerged in the liquid.

This then creates a dual effect – on the one hand the SAM molecules form the inner shell of the gas bubble, while on the other, they hinder free liquid motion on the film [2].

In foams with a small moisture content φ ($\varphi < 0.05$) the overwhelming portion of the liquid (depending on φ, up to 95%) is concentrated in unique formations called Plateau–Gibbs channels. These are formed at the joint between three films separating gas bubbles and recall in form a right cylinder having a Plateau triangle as its generatrix (a figure bounded by three pairs of tangent spheres of identical radius). Following a polyhedral model for the foam, we assume that the gas bubbles have the form of dodecahedra, while liquid content in the films and liquid displacement from channels into films and back can be neglected [1].

Considering the above, the process of propagation of an acoustical disturbance in foam can be represented in the following manner: Upon application of a pressure perturbation the gas bubbles change in size, which produces motion of the liquid contained in the foam. The liquid flow through a system of chaotically oriented capillaries (Plateau–Gibbs channels) is one of the hydrodynamic effects controlling evolution of the acoustical signal. The character of this flow will introduce certain unique features into the propagation of small amplitude oscillations.

In formulating the problem we will make a number of physical assumptions. We assume that the scale of the disturbances to be considered is such that it permits use of the linear acoustics approximation. We will limit our examination to only those foam motions for which the foam structure is not destroyed. We neglect the effects of gravity and assume the liquid to be weightless.

The foam density ρ_f can be written in the form:

$$\rho_f = \rho_1\varphi + \rho_2(1-\varphi). \tag{1}$$

Using the definition φ ($\varphi = V_1/(V_1 + V_2)$) and elementary relationships of the homogeneous model of [3] we can obtain an expression for the change in moisture content $\Delta\varphi$, considering the liquid volume V_1 and the gas volume V_2 to be independent variables. It should be recalled that the independence of V_1 implies that the problem is nonconservative. Physically, this means that liquid may enter the control volume under consideration as well as leave it. For the present problem we will limit ourselves to the conservative problem, i.e., assume that $\Delta V_1 \equiv 0$.

Assuming that the process is adiabatic, the change in bubble volume for $\Delta\varphi$ is given by:

$$\Delta\varphi = \varphi(1-\varphi)\frac{\Delta P_2}{\gamma P_{20}}. \tag{2}$$

It should be noted here that assumption of an adiabatic process is not required in principle. The problem of heat exchange between the gas in the bubble and the liquid in the Plateau–Gibbs channels is of undoubted scientific interest, and was considered in [4] using a cell scheme. However the goal of the present study is to investigate the effect of friction on liquid motion in the foam during sound propagation, so that the question of the character of interphase heat exchange will be reduced to choice of a value for the polytrophy index, which is arbitrary for the effects to be analyzed.

By variation of Eq. (1) with substitution of Eq. (2) for $\Delta\varphi$, with consideration that the gas state within the bubble is determined by a Mendeleev–Clapeyron equation, we obtain the equation of state of the foam in the linear approximation

$$\Delta\rho_f = \frac{\Delta P_2}{c_4^2};\ \frac{1}{c_4^2} = \frac{1}{c_0^2} + \frac{1}{c_3^2};\ c_0^2 = \frac{\gamma P_{20}}{\rho_1\varphi(1-\varphi)};\ c_3^2 = \frac{\gamma RT_{20}}{\mu_2(1-\varphi)}. \tag{3}$$

As in a continuous medium, the pressure P_f within the foam is a function of the gas pressure in the bubbles P_2, the pressure in the liquid channel P_1, and the moisture content φ. It was shown in [5] that for continuity of the medium and regularity of the functions used over the characteristic dimensions of the problem, the macroscopic parameters obtained by averaging over gas volumes and over surfaces within the phases coincide with each other. One consequence of this theorem is the fact that the volume and surface concentrations coincide at each point of the continuum at which the conditions of the theorem are satisfied. Consequently, for the pressure within the foam P_f one can use an expression similar to the one for density:

$$P_f = P_1\varphi + P_2(1-\varphi). \tag{4}$$

In writing the equation of motion for the foam it is necessary to consider the fact that the foam has an anomolously high viscosity [1], therefore the corresponding equation must include the frictional stress on the boundary.

Assuming that the foam volume under study is located within a cylindrical vessel of radius R_f, we can write the corresponding relationship in the form

$$\frac{\partial w}{\partial t} = -\frac{1}{\rho_f}\frac{\partial P_f}{\partial x} - \frac{\alpha_f}{\rho_f R_f}\tau_f, \tag{5}$$

where α_f is a coefficient defined by the form of the vessel section (in the case of a cylinder $\alpha_f = 2$).

The friction term in Eq. (5) τ_f depends significantly on the velocity profile in the foam flow. Because of inertia profile establishment requires a definite time, the ratio of which to the characteristic signal time determines the form of the function τ_f. In other words, the form of τ_f depends on how deep the viscous sublayer grows as compared to R_f. The thickness of the viscous sublayer δ_f is determined [6] by the signal carrier frequency ω and the kinematic viscosity of the foam ν_f: $\delta_f \simeq \sqrt{2\nu_f/\omega}$. Assuming for numerical estimates that $\nu_f \sim 10\nu_1$ [1], $R_f = 0.1$ m, we find that at $\nu > \nu_f^* = 2\nu_f/R_f^2 \simeq 2\cdot 10^{-3}$ sec^{-1} the viscous sublayer comprises a small fraction of the vessel radius R_f, i.e., for foam motion in the vessel under consideration a disturbance with frequency $\omega > \omega_f^*$ is a high frequency one. In this case the expression for τ_f can be obtained by reduction of the integral relationship and written in the form [6]:

$$\tau_f = \rho_f\sqrt{\frac{\nu_f}{\pi}}\int_{-\infty}^{t}\frac{\partial w}{\partial\tau}\frac{d\tau}{\sqrt{t-\tau}}. \tag{6}$$

Having written the continuity equation for the foam as a homogeneous medium in the usual form [6]

$$\frac{\partial\rho_f}{\partial t} + \frac{\partial(\rho_f w)}{\partial x} = 0 \tag{7}$$

and performing elementary substitutions and linearization, we eliminate the foam velocity w, and Eqs. (5)-(7) can be reduced to the following form:

$$\frac{\partial^2 P_f}{\partial x^2} = \frac{\partial^2\rho_f}{\partial t^2} - \frac{\alpha_f}{R_f}\sqrt{\frac{\nu_f}{\pi}}\int_{-\infty}^{t}\frac{\partial^2\rho_f}{\partial\tau^2}\frac{d\tau}{\sqrt{t-\tau}}. \tag{8}$$

Equation (8) is an equation of motion which considers friction on the boundary of foam contact with the vessel. It contains the transverse dimension of the vessel and can be used for processing of concrete experimental results, obtained, for example, with "shock tube" type devices [3].

One of the fundamental questions of foam acoustics is the relationship between the gas pressure in the bubbles P_2 and the liquid pressure in the channels P_1. It would be incorrect to use the Rayleigh equation in standard form [3, 5], since it is valid for an isolated bubble in an infinite liquid volume, while the bubbles are densely packed in the foam. Introduction of a correction for bubble interaction [5] is also not justifiable, since such corrections function at volume gas contents not exceeding 0.15-0.20, while in a foam this quantity lies in the range 0.95-1. A correct determination of the relationship between P_2 and P_1 requires consideration of the basic mechanisms because of which the difference $P_2 - P_1$ is nonzero surface tension, liquid motion, the combined mass effect, and the viscous deformed boundary. We will consider these in somewhat more detail while keeping in mind the quasi-ordered structure of the foam.

It follows from Plateau's experimental data [1] and general physical considerations [7] that at each corner of the dodecahedron there are four Plateau–Gibbs channels of which three are edges of the given polyhedron, while the fourth is oriented normally, if that term can be applied to a dodecahedron, to the surface of the foam cell structure under consideration. With change in the mean dimension of the foam cell (for concreteness, with reduction in external pressure) the liquid as it were flows into this "foreign" channel from the center of the fixed gas bubble, while its "native" channels with the liquid they contain play the role of combined mass for this cell. Consequently, the dynamics of liquid displacement in the "foreign" channel are practically determined by the dependence of bubble radius on time $r_b(t)$ and the degree of freedom of liquid motion in the foam. The mathematical equivalent of the latter concept is the hydroconductivity K_f – a function of the physical parameters of the foam: the dispersion r_b and the moisture content φ [8].

Commencing from the above, for the liquid phase of the system we write equations of the following form:

$$\frac{\partial}{\partial r}(r^2\varphi u) = 0,$$

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho_1}\frac{\partial P_1}{\partial r} - \nu_1\varphi\frac{u}{K_f}. \tag{9}$$

Here the momentum transport equation is taken in the Brinkman form – a unique superposition of the Darcy and Navier–Stokes equations [9]. The term linear in the liquid velocity u corresponding to dissipation due to penetration of liquid into the "foreign" channel, imposes limitations on the range of applicability of the set of Eqs. (9).

This is because friction linear in velocity indicates the presence in the channel of a Poiseuille flow regime. This in turn implies that the viscous sublayer in the Plateau–Gibbs channel grows in size and occupies the entire channel volume, i.e., the sublayer thickness δ_1 is larger than the characteristic channel transverse dimension, which we may denote by $\sqrt{S}$.

Thus, the notation of Eq. (9) implies quasisteady liquid flow in the channel, which limits the upper frequency of model applicabililiy:

$$\sqrt{S} < \delta_1 = \sqrt{\frac{2\nu_1}{\omega}} \Rightarrow \omega < \omega_1^* = \frac{2\nu_1}{S}.$$

The equations of system (9) must be integrated over r from r_b to $r_b + b_0$, with b_0 determined from the condition of smallness of $\varphi(\varphi \approx 3b_0/r_b)$. On the bubble surface boundary conditions of the normal form must be satisfied [3, 5, 6]:

$$u = \dot{r}_b,\ \ P_1 = P_2 - \frac{2\sigma}{r_b} - 4\mu_1\frac{\dot{r}_b}{r_b}\ \ (\mu_1 = \rho_1\nu_1). \tag{10}$$

In writing Eq. (10) we have neglected the difference in capillary pressure determined by the bubble pressure ($2\sigma/r_b$) and the lateral surface of the Plateau–Gibbs channel ($\sigma/(\alpha_1\sqrt{S})$, $\alpha_1^{-2} = \pi/2 - \sqrt{3}$) [1]. After integration of Eq. (9) with conditions (10) we obtain the equation:

$$\rho_1 b_0 \ddot{r}_b + \mu_1\left[4 + \frac{\varphi r_b^2}{K_f}\right]\frac{\dot{r}_b}{r_b} + \frac{2\sigma}{r_b} = P_2 - P_1, \tag{11}$$

which is an analog of the Rayleigh equation for a foam within the framework of the proposed model.

To determine the relationship between Δr_b and ΔP_2 we make use of the previously introduced assumption of adiabatic change in bubble volume, used in deriving Eq. (2). In this case $\Delta r_b/r_b = -\Delta P_2/(3\gamma P_{20})$. Substituting this expression in Eq. (11) we obtain the relationship between P_1 and P_2 in the form

$$-\rho_1\frac{r_b^2\varphi}{9\gamma P_{20}}\ddot{P}_2-\frac{\mu_1 r_b}{3\gamma P_{20}}\left[4+\frac{\varphi r_b^2}{K_f}\right]\frac{\dot{P}_2}{r_b}+\frac{2\sigma}{r_b}=P_2-P_1. \quad (12)$$

Turning to deviations from the steady state values, we have the following expression:

$$-\rho_1\frac{r_b^2\varphi}{9\gamma P_{20}}\ddot{P}_2-\frac{\mu_1 r_b}{3\gamma P_{20}}\left[4+\frac{\varphi r_b^2}{K_f}\right]\frac{\dot{P}_2}{r_b}+\frac{2\sigma}{r_b}\frac{\Delta P_2}{3\gamma P_{20}}=\Delta P_2-\Delta P_1. \quad (13)$$

Expressing ΔP_1 from Eq. (13) in terms of ΔP_2, $\dot{P}_2$, $\ddot{P}_2$ and substituting the expressions obtained in Eq. (4), we find the form of the dependence of change in foam pressure ΔP_f on deviation of the pressure in the gas and its two first derivatives. Introducing the function $\Delta P_f = \Delta P_f(\Delta P_2, \dot{P}_2, \ddot{P}_2)$ thus obtained $\Delta\rho_f$ into the left side of the equation of motion Eq. (8), and the fraction $\Delta P_2/c_4{}^2$ from the equation of state of the foam Eq. (3) on the right side, we obtain a quasilinear wave equation in the form

$$P_{2tt}-c_4^2P_{2xx}\left\{1-\frac{2\sigma}{r_b}\frac{1+3\varphi(1-\varphi)}{3\gamma P_{20}}\right\}-\frac{\alpha_f}{R_f}\sqrt{\frac{\nu_f}{\pi}}\int_{-\infty}^{t}\frac{\partial^2 P_2}{\partial\tau^2}\frac{d\tau}{\sqrt{t-\tau}}-$$

$$-c_4^2\varphi\frac{\partial^2}{\partial x^2}\left\{\frac{\mu_1}{3\gamma P_{20}}\left[4+\frac{\varphi r_b^2}{K_f}\right]P_{2t}+\rho_1\frac{r_b^2\varphi}{9\gamma P_{20}}P_{2tt}\right\}=0. \quad (14)$$

Equation (14) contains solutions corresponding to waves propagating in both directions. After "changing" the derivative, i.e., using the approximation $\partial/\partial t = -c_4(\partial/\partial x)$, the equation can be reduced to a form corresponding to wave displacement in one direction (to the form of the Burgers—Corteweg—de Brise equations). However, to obtain and study the dispersion relationship it is desirable to maintain the form of Eq. (14).

The dispersion relationship corresponding to wave equation (14) can be written in the form

$$k^2=\frac{\omega^2}{c_4^2}f(\omega)\left\{1-\frac{2\sigma}{r_b}\frac{1+3\varphi(1-\varphi)}{3\gamma P_{20}}-\right.$$

$$\left.-i\omega\frac{\mu_1\varphi}{3\gamma P_{20}r_b}\left(4+\frac{\varphi r_b^2}{K_f}\right)-\omega^2\rho_1\frac{r_b^2\varphi^2}{9\gamma P_{20}}\right\}^{-1},\quad f(\omega)=1+\alpha_f(1+i)\sqrt{\frac{\omega_f^*}{\omega}}. \quad (15)$$

Analysis of dispersion relationship (15) can be commenced reasonably by defining the basic characteristics of the model which are determined by the foam parameters chosen. We will consider a foam with moisture content $\varphi = 0.01$, and dispersion (cell size) $r_b = 10^{-3}$ m. In this case the characteristic cross section of a Plateau—Gibbs channel proves to be $S \simeq 10^{-8}$ m^2, which gives an upper limit for model applicability of $\omega_1{}^* \simeq 10^2$ sec^{-1}. Assuming that the characteristic dimension of the vessel holding the foam comprises $R_f \simeq 0.1$ m, we obtain for the quantity $\omega_f{}^*$ the numerical value 10^{-3} sec^{-1}. We will study Eq. (15) with consideration of these estimates.

We begin by analyzing the effect of surface tension, which leads to an insignificant reduction in phase velocity. The scale of this reduction can easily be estimated by considering that the presence of SAM reduces the surface tension coefficient to a value of $\sigma = 0.032$ N/m [1]. In this case the ratio of the capillary pressure $2\sigma/r_b$ to atmospheric (assuming normal conditions) does not exceed 10^{-3}.

It is obvious that such a contribution to dispersion may be neglected. An estimate of the action of the combined mass effect can also be easily done. The small content of liquid in the foam leads to a decrease in combined mass per single gas bubble, as compared to the analogous situation with close to unity moisture content [3, 5].

From this follows, in particular, the appearance in the first term on the left side of Eq. (11) of the factor b_0 in place of the r_b of the usual Rayleigh equation [3, 5], and further, the shift in the resonant bubble frequency to the right on the frequency scale. Since we are operating in the low frequency range, and the range of applicability of the model relative to the effect of this mechanism will be determined by the expression $\rho_1\varphi r_b{}^2\omega^2/(9\gamma P_{20})$, which at $\omega = 10^2$ sec^{-1} (the upper frequency limit) and the foam parameters chosen will equal approximately 10^{-10}. Naturally, such a contribution may be neglected completely.

The contribution to sound propagation in the foam of the viscous boundary can be examined by considering liquid motion in the Plateau–Gibbs channels. It is appropriate to note here that there is at present no unified opinion regarding the function $K_f(\varphi;\ r_b)$. This is related to the experimental difficulty of observing liquid motion in the foam. This is true because even in a foam with structure very close to polyhedral ($\varphi < 0.01$), application of a pressure disturbance leads to liquid flow over the films forming the polyhedron faces, while it is impossible to experimentally determine the fraction of liquid which flows in that manner (as contrasted to that which flows through channels). We will choose the function K_f in the Koseni–Karman form [5] with the numerical coefficient presented in [8]. This choice was based on the fact that the expression for the permittivity coefficient obtained upon transition from Brinkman equation (9) to the Darcy equation then describes the results of experiments performed by Kann [8] in porous media at the heat-mass transport laboratory of the Northern Exploitation Problems Institute, Siberian Branch, Academy of Sciences of the USSR.

With the notation used herein the function $K_f(\varphi;\ r_b)$ appears as:

$$K_f(\varphi;\ r_b) = 3{,}48\cdot 10^{-3} r_b^2\, \varphi^3. \tag{16}$$

Substituting Eq. (16) in the third term of the denominator of Eq. (15), we find the ratio $1{:}10^6$ for comparison of the viscous boundary mechanism and liquid motion through channels. It is obvious that to the accuracy used herein the action of viscous boundary effects may be neglected.

Analysis of the effect of friction on the surface of the vessel containing the foam reduces to consideration of the function $f(\omega)$, more precisely, the radical contained therein. Assuming a cylindrical vessel $\alpha_f = 2$ the intensity of the friction is determined by a radical of the form $\sqrt{\omega_f^*/\omega}$. Near the left edge of the range of model applicability the value of the radical is close to unity, but with motion upward in frequency it decreases to the order of 10^{-2}.

It is understandable that the contribution of such a quantity cannot be neglected and we must retain the function $f(\omega)$ without changes.

Thus, analytical study of dispersion equation (15) shows that of all the physical mechanisms analyzed the dominant ones are effects related to friction on the gas–liquid and foam–solid boundaries. With consideration of this fact Eq. (15) can be written in the simpler form

$$k^2 = \frac{\omega^2}{c_4^2} f(\omega)\left[1 - i\omega\,\frac{\mu_1\varphi^2}{3\gamma P_{20}K_f}\right]^{-1}, \tag{17}$$

while Eq. (14) reduces to a classical wave equation of the form $u_{tt} - c_4{}^2 u_{xx} = 0$ with dissipative right side in the form of the third derivative (Burgers term) and quasilinear term in the form of a Duhamel integral [3]:

$$P_{2tt} - c_4^2 P_{2xx} = c_4^2\,\frac{\varphi^2\mu_1}{3\gamma P_{20}K_f}\,P_{2xxt} + \frac{\alpha_f}{R_f}\sqrt{\frac{\nu_f}{\pi}}\int\limits_{-\infty}^{t}\frac{\partial^2 P_2}{\partial\tau^2}\,\frac{d\tau}{\sqrt{t-\tau}} = 0. \tag{18}$$

By "transforming" the derivative Eq. (18) can be reduced to a linear Burgers equation with quasilinear right side in the form of the convolution

$$P_t + c_4 P_x - c_4^2\,\frac{\varphi^2\mu_1}{3\gamma P_{20}K_f}\,P_{xx} = \frac{\alpha_f}{R_f}\sqrt{\frac{\nu_f}{\pi}}\int\limits_{-\infty}^{t}\frac{\partial P}{\partial\tau}\,\frac{d\tau}{\sqrt{t-\tau}}.$$

We will consider the dispersion relationship (17) assuming smallness of the imaginary term in square brackets and the radical in the function $f(\omega)$. In this case Eq. (17) can be expanded in a small parameter and we obtain an approxiamtion of Eq. (15) in the form

$$k^2 = \frac{\omega^2}{c_4^2}\left[1 + (1+i)\sqrt{\frac{\omega_f^*}{\omega}} + \frac{i\omega}{\omega_0}\right];\quad \omega_0 = \frac{10{,}44\gamma P_{20}\varphi}{10^3\mu_1}. \tag{19}$$

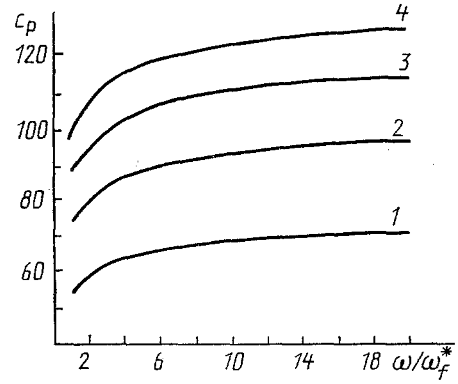

Fig. 1

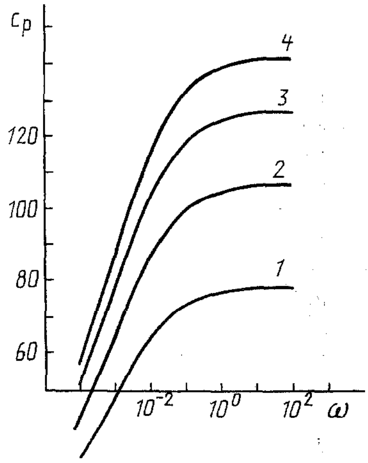

Fig. 2

Fig. 1. Phase velocity dispersion near lefthand boundary of frequency range, $r_b = 10^{-3}$ m: 1) $\varphi = 0.02$; 2) 0.01; 3) 1/150; 4) 0.005. c_p, m/sec.

Fig. 2. Function $c_p(\omega)$ calculated with Eq. (19) for foam with parameters: $r_b = 10^{-3}$ m: 1) $\varphi = 0.02$; 2) 0.01; 3) 1/150; 4) 0.005.

The value of ω_0 for the problem parameters chosen comprises 10^4 sec^{-1}. Elementary analysis of Eq. (19) shows that the dispersion of the phase velocity falls radically with frequency and is caused by friction on the foam—solid phase boundary, with the function $c_p(\omega)$ being representable in the form:

$$c_p(\omega) = \frac{\omega}{\mathrm{Re}\,[k(\omega)]} = \frac{c_4}{1+\sqrt{\frac{\omega_f^*}{\omega}}}. \tag{20}$$

Figure 1 shows graphs of the function $c_p(\omega)$ near the lefthand boundary of the frequency range (frequency ω_f^*). With increase in frequency the effect of the radical $\sqrt{\omega_f^*/\omega}$ decreases and in the limit $\omega \to \omega_1^*$ the value of $c_p(\omega)$ tends to the quantity c_4, which defines the propagation rate of disturbances in the gas—liquid medium [3], since friction on the interphase surface dies but contributes to phase velocity dispersion in accordance with Eq. (20). The curves of $c_p(\omega)$ shown in Fig. 2, calculated with Eq. (19) for the various physical parameters characterizing the foam, indicate that near the right side of the range of applicability the propagation rate for small amplitude oscillations in the foam is defined with good accuracy by the quantity c_4.

The attenuation decrement of an acoustical signal, defined by the imaginary part of the wave vector $\mathrm{Im}[k(\omega)]$, is composed of two parts, caused by friction during motion of the foam as a whole in the vessel, and friction during liquid flow in the system of Plateau—Gibbs channels. Expanding Eq. (19) in a small parameter, we obtain the attenuation decrement in the form of the sum of two terms

$$\mathrm{Im}\,[k(\omega)] = \frac{1}{2c_4}\left[\sqrt{\omega\omega_f^*} + \frac{\omega^2}{\omega_0}\right]. \tag{21}$$

The contribution from motion of the foam as a whole is proportional to the root of the frequency and is a slow frequency of ω. But the effect of liquid motion through channels is described by a quadratic function of frequency.

Consequently, there exists a critical frequency ω_*, at which the dominant mechanism of sound wave energy dissipation changes. The value is easily determined: $\omega_* = (\omega_f^*/\omega_0{}^2)^{1/3}$. It can easily be seen that in the interval $\omega_f^* < \omega < \omega_*$ attenuation is determined mainly by the effective foam viscosity ν_f and the vessel radius R_f, while in the range $\omega_* < \omega < \omega_1^*$ the attenuation decrement is proportional to the liquid viscosity μ_1 and inversely proportional to the moisture content φ and the initial gas pressure in the bubbles P_{20}. Figure 3

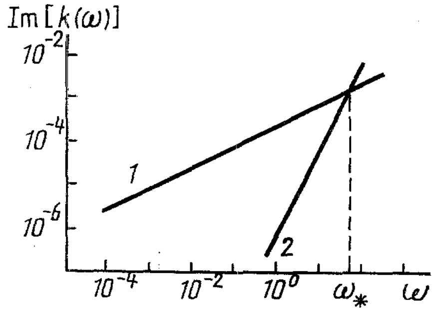

Fig. 3

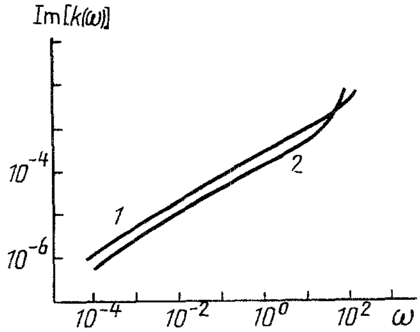

Fig. 4

Fig. 3. Attenuation decrement produced by: 1) foam friction on vessel wall; 2) friction on interphase boundary.

Fig. 4. Total attenuation decrement calculated from dispersion equation (17): 1) $r_b = 10^{-3}$ m, $\varphi = 0.02$; 2) $r_b = 10^{-3}$ m, $\varphi = 0.005$.

shows curves of each of the two terms of Eq. (21) in logarithmic scale and notes the point of their intersection (ω_*). Numerical calculation of ω_* for the foam parameters used yields a value of the order of $5 \cdot 10^1$ sec^{-1}, i.e., over the greater part of the interval on the frequency scale over which the proposed model is valid, the dominant dissipation mechanism is friction on the vessel wall. On the other hand, the effect of liquid friction dominates over a narrower, but practically more interesting range.

Analysis of the imaginary component of the wave vector $k(\omega)$ would be incomplete if we did not consider one unique feature of Eq. (21): within the framework of the proposed model dissipation does not depend on the dispersion of the foam, i.e., the gas bubble radius r_b does not appear in the final expression. This is explained by the form chosen for the function $K_f(\varphi; r_b)$, which has the physical meaning of hydroconductivity [8]. The dependence of the decrement on foam structural parameters reduces to that on the moisture content φ, contained in the denominator of the term corresponding to the effect of liquid microflows. However, it should be noted that this dependence is quite strong. This is demonstrated by Fig. 4, which shows attenuation decrements calculated for the complete Eq. (17). What is at first a slight difference between the curves becomes weighty when we note that the figure is drawn in logarithmic scale. The difference in the attenuation coefficients in the range $\omega_f < \omega < \omega_*$ is explained by the factor c_4, the effect of this cofactor being especially clear in Eq. (21). In the interval $\omega_* < \omega < \omega_1{}^*$ the dependence of the decrement on φ is determined by the combined contribution of the velocity c_4 which depends on the average moisture content, and the factor φ itself.

Thus the dominant influence on propagation of low frequency sound in a gas–liquid foam is produced by mechanisms related to friction on the foam–solid boundary and on interphase boundaries.

Notation. φ, foam moisture content; ρ, density, kg/m^3; V, volume, m^3; P, pressure, Pa; γ, adiabatic index of gas; c_0, "frozen" speed of sound in two-phase mixture, m/sec; c_3, speed of sound in gas, m/sec; μ_2, molecular weight of gas, kg/mol; R_f, vessel radius, m; τ, frictional stress, Pa; ν, kinematic viscosity, m^2/sec; ω, frequency, sec^{-1}; w, foam velocity, m/sec; u, liquid velocity, m/sec; r, radius, m; S, Plateau–Gibbs channel section, m^2; σ, surface tension coefficient, N/m; k, wave vector, m^{-1}; x, coordinate, m; t, time, sec. Subscripts: 1, liquid; 2, gas; f, foam as a whole; b, individual bubble. A dot above a variable denotes differentiation with respect to time.

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WAVE FLOW OF VISCOELASTIC SUSPENSIONS IN TUBES

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The wave propagation of viscoelastic suspensions in tubes is theoretically investigated, taking account of the deformation of the tube walls and the dispersity of the medium within the framework of the rheological model of a viscoelastic liquid with internal oscillators. A wave equation is obtained, and its limiting cases are analyzed. The dispersional relation is investigated with characteristic values of the rheological parameters of the medium. A numerical experiment is undertaken to investigate the influence of the rheology of the medium on the structure and dynamics of wave perturbation of velocity perturbations.

Introduction. The increased production of anomalous petroleum has prompted the active investigation of rheophysical problems of oil and gas production [1]. The high content of paraffin, naphthene, and aromatic hydrocarbons in the petroleum extracted and transported, which are present in the form of solid-phase disperse particles at the certain temperatures, means that the solid-hydrocarbon content may reach 18-20%. This leads to various anomalies in the rheodynamic properties of petroleum and hydrodynamic peculiarities in pipeline transport [2-3]. In particular, nonsteady wave conditions of flow appear in pipeline startup, with variation in pumping-station operating conditions, in emergency situations, etc. Experimental investigation of shock-wave propagation in paraffin petroleum and modeling of such media [4] shows the presence of new, previously undescribed features in the propagation of waves in petroleum. It is found that increase in solid-particle concentration leads to significant distortion of the structure and dynamics of shock-wave propagation. The distortion is such that it cannot be described within the framework of existing models of viscoelastic liquids [5]. In connection with this, there is a need to investigate the influence of rheological properties of suspensions on wave processes on the basis of fundamentally new models. The possibility of using the model of viscoelasticity with internal oscillators is considered below [6].

Since anomalous petroleum has viscoelastic properties [3], it is fairly difficult to determine the parameters of interphase interaction of such materials with disperse solid particles and hence to describe the media within the framework of a multispeed continuum. At the same time, taking into account that the densities of the liquid and solid phases are similar, and the particle dimensions are many times less than the distances between them, the tube diameters, and the given wavelengths, it is expedient to model the medium as quasi-homogeneous, neglecting the dynamic and inertial effects in the relative motion of the components. However, in this case, the medium is assumed to be continuous, and the presence of solid particles is only indirectly taken into account: by the change in rheological constants as a function of the concentration. This assumption of continuity of the medium eliminates the possibility of taking direct account of the influence of the particle dynamics on the wave propagation at a wavelength much greater than the particle size.

The problem of taking account of the dynamics of solid particles in a viscoelastic medium is analogous to that which arises in considering problems of nonlinear seismics [6]. On the basis of the analysis of experimental data on wave propagation in quartz and of various

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